Vector Algebra

Question1

Two vectors $a\hat{i}+b\hat{j}+\hat{k}$ and $2\hat{i}-3\hat{j}+4\hat{k}$ are perpendicular to each other. When $3\mathrm{a}+2\mathrm{~b}=7$, the ratio of a to b is $\frac{x}{2}$. The value of x is

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Options:

A. zero

B. 2

C. 1

D. 4

Answer: C

Solution:

$\vec{A}=a\hat{i}+b\hat{j}+\hat{k} ; \vec{B}=2\hat{i}-3\hat{j}+4\hat{k}$

As the two vectors are perpendicular to each other

$\therefore \quad \vec{A}\cdot\vec{B}=0$

$\therefore \quad 2a-3b+4=0$

$3a+2b=7 \quad$ ....(given)

Solving for a and b , we get

$a=1, b=2$

$\therefore \quad a:b=1:2$

$\therefore$ value of x is 1

Question2

The vector sum of two forces $\vec{A}$ and $\vec{B}$ is perpendicular to their vector difference. Hence forces $\vec{A}$ and $\vec{B}$ are

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Options:

A.

perpendicular to each other.

B.

parallel to each other.

C.

unequal in magnitude.

D.

equal in magnitude.

Answer: D

Solution:

The sum of the two forces,

$$\overrightarrow{F_1} = \vec{A} + \vec{B} \qquad \ldots(i)$$

The difference of the two forces,

$$\vec{F}_2 = \vec{A} - \vec{B} \qquad \ldots(ii)$$

Since sum of the two forces is perpendicular to their difference,

$$\begin{aligned} & \vec{F}_1 \cdot \vec{F}_2 = 0 \\ \Rightarrow\ & (\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0 \\ \Rightarrow\ & A^2 - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - B^2 = 0 \\ \therefore\ & A^2 = B^2 \Rightarrow |A| = |B| \end{aligned}$$

Thus, the forces are equal to each other in magnitude.

Question3

If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$
$|\vec{a} \times \vec{b}| = 35$, find $\vec{a} \cdot \vec{b}$

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Options:

A. 4

B. 5

C. 6

D. 7

Answer: D

Solution:

$$|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$$
$$|\vec{a} \times \vec{b}| = 35$$
$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$$
$$\sin \theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$$
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$$
$$= \sqrt{26} \cdot 7 \cdot \cos \theta$$
$$\sin \theta = \frac{5}{\sqrt{26}}$$
$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{25}{26}} = \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}}$$
$$\vec{a} \cdot \vec{b} = \sqrt{26} \cdot 7 \cdot \frac{1}{\sqrt{26}} = 7$$

Question4

Vector $\vec{A}$ of magnitude $5\sqrt{3}$ units, another vector $\vec{B}$ of magnitude of 10 units are inclined to each other at an angle of 30°. The magnitude of vector product of the two vectors is $\left[\sin 30^\circ = \frac{1}{2}\right]$

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Options:

A. $5\sqrt{3}$ units

B. 10 units

C. $25\sqrt{3}$ units

D. 75 units

Answer: C

Solution:

We have

$|\vec{A}| = 5\sqrt{3}, \quad |\vec{B}| = 10, \quad \theta = 30^\circ.$

The magnitude of the cross product is

$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\sin\theta.$

Substitute the values:

$|\vec{A} \times \vec{B}| = (5\sqrt{3})(10)(\sin 30^\circ).$

$= 50\sqrt{3} \cdot \frac{1}{2}$

$= 25\sqrt{3}.$

✅ Correct Answer: **Option C: $25\sqrt{3}$ units**

Question5

If $\vec{P} = b\hat{i} + 6\hat{j} + \hat{k}$ and $\vec{Q} = \hat{i} - a\hat{j} + 4\hat{k}$ are perpendicular to each other, also $3\text{ b} - \text{a} = 5$. The value of a and b is

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Options:

A. $\mathrm{a} = 2,\ \mathrm{b} = 10$

B. $\mathrm{a} = 1,\ \mathrm{b} = 2$

C. $\mathrm{a} = 2,\ \mathrm{b} = 3$

D. $\mathrm{a} = 4,\ \mathrm{b} = 3$

Answer: B

Solution:

We are given vectors:

$\vec{P} = b\hat{i} + 6\hat{j} + \hat{k}, \quad \vec{Q} = \hat{i} - a\hat{j} + 4\hat{k}.$

Step 1: Condition for perpendicularity

$\vec{P} \cdot \vec{Q} = 0$

$(b)(1) + (6)(-a) + (1)(4) = 0$

$b - 6a + 4 = 0 \quad \Rightarrow \quad b - 6a = -4 \quad \Rightarrow \quad b = 6a - 4$

Step 2: Given relation

$3b - a = 5$

Substitute $b = 6a - 4$:

$3(6a - 4) - a = 5$

$18a - 12 - a = 5$

$17a - 12 = 5$

$17a = 17 \quad \Rightarrow \quad a = 1$

Step 3: Solve for b

$b = 6(1) - 4 = 2$

✅ **Final Answer:**

$a = 1,\ b = 2$

Correct Option: B

Question6

Given $\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k})$, $\vec{B} = (3\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{C} = (3\hat{i} + 2\hat{j} + \hat{k}) \cdot (\vec{A} + \vec{B}) \cdot \vec{C}$ will be

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Options:

A. 10

B. 12 c

C. 18

D. 20

Answer: A

Solution:

$$\begin{aligned}
&\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k}) \\
&\vec{B} = (3\hat{i} + \hat{j} - 2\hat{k}) \\
&\vec{C} = (3\hat{i} + 2\hat{j} + \hat{k}) \\
&\vec{A} + \vec{B} = (2+3)\hat{i} + (-3+1)\hat{j} + (1-2)\hat{k} \\
&\qquad\quad = 5\hat{i} - 2\hat{j} - \hat{k} \\
&(\vec{A} + \vec{B}) \cdot \vec{C} = (5\hat{i} - 2\hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} + \hat{k}) \\
&\qquad\qquad\quad = 15 - 4 - 1 \\
&\qquad\qquad\quad = 10
\end{aligned}$$

Question7

Given $\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k})$, $\vec{B} = (3\hat{i} + \hat{j} - 2\hat{k})$ and $\vec{C} = (3\hat{i} + 2\hat{j} + \hat{k}) \cdot (\vec{A} + \vec{B}) \cdot \vec{C}$ will be

Options:

A. 10

B. 12 c

C. 18

D. 20

Answer: A

Solution:

We're asked to compute something involving vectors:

$$\vec{A} = (2\hat{i} - 3\hat{j} + \hat{k}), \quad \vec{B} = (3\hat{i} + \hat{j} - 2\hat{k}), \quad \vec{C} = (3\hat{i} + 2\hat{j} + \hat{k}).$$

Expression: $(\vec{A} + \vec{B}) \cdot \vec{C}$.

Step 1. Add A and B:

$$\vec{A} + \vec{B} = (2+3)\hat{i} + (-3+1)\hat{j} + (1-2)\hat{k} = 5\hat{i} - 2\hat{j} - \hat{k}.$$

So $\vec{A} + \vec{B} = (5, -2, -1)$.

$\vec{C} = (3, 2, 1)$.

Step 2. Dot product:

$$(\vec{A} + \vec{B}) \cdot \vec{C} = (5)(3) + (-2)(2) + (-1)(1) = 15 - 4 - 1 = 10.$$

✅ **Final Answer:**

$$(\vec{A} + \vec{B}) \cdot \vec{C} = 10$$

So the correct option is **A. 10**.

Question8

A unit vector in the direction of resultant vector of $\vec{A} = -2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 4\hat{k}$ is

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Options:

A. $\frac{-3\hat{\mathrm{i}}+\hat{\mathrm{j}}+5\hat{\mathrm{k}}}{\sqrt{35}}$

B. $\frac{\hat{i}+2\hat{j}-4\hat{k}}{\sqrt{35}}$

C. $\frac{-2\hat{\mathrm{i}}+3\hat{\mathrm{j}}+\hat{\mathrm{k}}}{\sqrt{35}}$

D. $\frac{-\hat{\mathrm{i}}+5\hat{\mathrm{j}}-3\hat{\mathrm{k}}}{\sqrt{35}}$

Answer: A

Solution:

We are asked:

A unit vector in the direction of the resultant vector of

$\vec{A} = -2\hat{i} + 3\hat{j} + \hat{k}, \quad \vec{B} = \hat{i} + 2\hat{j} - 4\hat{k}.$

Step 1: Find the resultant

$\vec{R} = \vec{A} + \vec{B} = (-2+1)\hat{i} + (3+2)\hat{j} + (1-4)\hat{k} = (-1)\hat{i} + 5\hat{j} - 3\hat{k}.$

So,

$\vec{R} = -\hat{i} + 5\hat{j} - 3\hat{k}.$

Step 2: Magnitude of $\vec{R}$

$|\vec{R}| = \sqrt{(-1)^2 + 5^2 + (-3)^2} = \sqrt{1+25+9} = \sqrt{35}.$

Step 3: Unit vector in the direction of $\vec{R}$

$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{-\hat{i}+5\hat{j}-3\hat{k}}{\sqrt{35}}.$

✅ **Final Answer:**

Option D

$\frac{-\hat{i}+5\hat{j}-3\hat{k}}{\sqrt{35}}$

Question9

The three vector $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}, \vec{B} = \hat{i} - 3\hat{j} + 5k$ and $\vec{C} = 2\hat{i} - \hat{j} + 4\hat{k}$ will form

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Options:

A. isosceles triangle.

B. equilateral triangle.

C. no triangle.

D. right angled triangle.

Answer: C

Solution:

$\overrightarrow{\mathrm{A}} = 3\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$

$\overrightarrow{\mathrm{B}} = \hat{\mathrm{i}} - 3\hat{\mathrm{j}} + 5\mathrm{k}$

$\overrightarrow{\mathrm{C}} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$

$|\mathrm{A}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

$|\mathrm{B}| = \sqrt{1^2 + (-3)^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$

$|\mathrm{C}| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$

$\therefore$ The three vectors do not form an equilateral or isosceles triangle because none of the sides have equal lengths.

$$\begin{aligned}\vec{A} \cdot \vec{B} &= (3)(1) + (-2)(-3) + (1)(5) \\ &= 3 + 6 + 5 = 14\end{aligned}$$

$$\begin{aligned}\overrightarrow{\mathrm{B}} \cdot \overrightarrow{\mathrm{C}} &= (1)(2) + (-3)(-1) + (5)(4) \\ &= 2 + 3 + 20 = 25\end{aligned}$$

$$\begin{aligned}\overrightarrow{\mathrm{A}} \cdot \overrightarrow{\mathrm{C}} &= (3)(2) + (-2)(-1) + (1)(4) \\ &= 6 + 2 + 4 = 12\end{aligned}$$

None of the sides of the triangle are perpendicular to each other, as the dot product of any pair of vectors is not zero.

If the vectors are coplanar, then they will form a triangle (unless any one of them is parallel to each other)

For three vectors to be coplanar, scalar triple product should be zero.

$$A \cdot (B \times C) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 3 & -2 & 1 \\ 1 & -3 & 5 \\ 2 & -1 & 4 \end{vmatrix} = -28$$

Since the scalar triple product is non-zero (-28), the vectors A, B, and C are not coplanar and hence do not form a triangle.

Question10

Three vectors are expressed as $\vec{a} = 4\hat{i} - \hat{j}, \vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$. The unit vector along the direction of sum of these vectors is

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Options:

A. $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$

C. $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}+\hat{k})$

D. $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

Answer: A

Solution:

$$\begin{aligned} \vec{r} &= \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k} \\ &= \hat{i} + \hat{j} - \hat{k} \\ \hat{r} &= \frac{\vec{r}}{|r|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} \end{aligned}$$

Question11

If $\vec{A} = \hat{i} + \hat{j} + 3\hat{k}, \vec{B} = -\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{C} = 2\hat{i} - 2\hat{j} - 8\hat{k}$, then the angle between the vectors $\overrightarrow{\mathrm{P}} = \overrightarrow{\mathrm{A}} + \overrightarrow{\mathrm{B}} + \overrightarrow{\mathrm{C}}$ and $\overrightarrow{\mathrm{Q}} = (\overrightarrow{\mathrm{A}} \times \overrightarrow{\mathrm{B}})$ is (in degree)

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Options:

A. 0°

B. 45°

C. 90°

D. 60°

Answer: C

Solution:

$$\begin{aligned} \vec{P} &= \vec{A} + \vec{B} + \vec{C} \\ &= \hat{i} + \hat{j} + 3\hat{k} - \hat{i} + \hat{j} + 4\hat{k} + 2\hat{i} - 2\hat{j} - 8\hat{k} \end{aligned}$$

$$\overrightarrow{\mathrm{P}} = 2\hat{\mathrm{i}} - \hat{\mathrm{k}}$$

$$\begin{aligned} \overrightarrow{\mathrm{Q}} = \overrightarrow{\mathrm{A}} \times \overrightarrow{\mathrm{B}} &= \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 1 & 1 & 3 \\ -1 & 1 & 4 \end{vmatrix} \\ &= \hat{\mathrm{i}}(4-3) - \hat{\mathrm{j}}(4+3) + \hat{\mathrm{k}}(1+1) \end{aligned}$$

$$\overrightarrow{\mathrm{Q}} = \hat{\mathrm{i}} - 7\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$$

Angle between $\vec{P}$ and $\vec{Q}$ is given by

$$\begin{aligned} \cos\theta &= \frac{\overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{Q}}}{|\overrightarrow{\mathrm{P}}||\overrightarrow{\mathrm{Q}}|} \\ &= \frac{(2\hat{\mathrm{i}} - \hat{\mathrm{k}}) \cdot (\hat{\mathrm{i}} - 7\hat{\mathrm{j}} + 2\hat{\mathrm{k}})}{\sqrt{(2)^2 + (0)^2 + (1)^2} \cdot \sqrt{(1)^2 + (7)^2 + (2)^2}} \\ \therefore \quad \cos\theta &= 0 \mathrm{~S} \\ \therefore \quad \theta &= 90 \end{aligned}$$

Question12

The resultant of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{C}$. If the magnitude of $\vec{B}$ is doubled, the new resultant vector becomes perpendicular to $\vec{A}$, then the magnitude of $\overrightarrow{\mathrm{C}}$ is

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Options:

A. 4 B

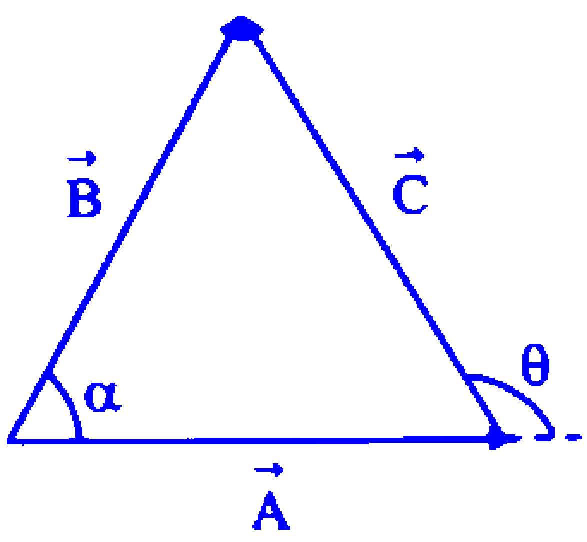
B. 3 B

C. B

D. 2 B

Answer: C

Solution:



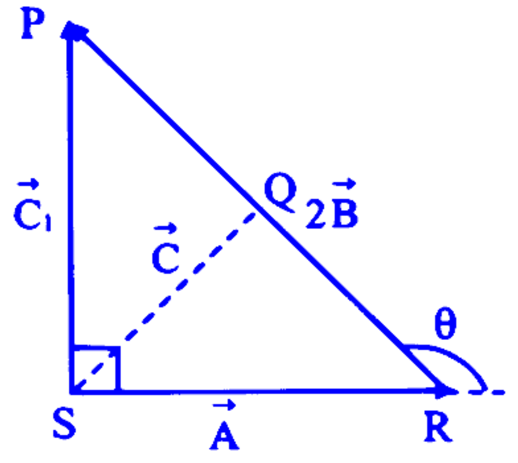
$\vec{A}+\vec{B}=\vec{C} \quad \vec{A}$

$C^2=A^2+B^2+2AB\cos\theta \qquad \ldots(i)$

and $\tan \alpha = \frac{\mathrm{B} \sin \theta}{\mathrm{A}+\mathrm{B} \cos \theta}$

When B is doubled, resultant is perpendicular to $\vec{A}$

$\therefore C_1^2 = A^2 + 4B^2 + 4AB \cos \theta \qquad \ldots (ii)$



From right angled triangle PSR

$(2B)^2 = C_1^2 + A^2$
$C_1^2 = 4B^2 - A^2$

Substituting in (ii) and solving,

$\mathrm{A}^2 + 2\mathrm{AB} \cos \theta = 0 \qquad \ldots (iii)$

Substituting (iii) in (i), $\mathrm{C} = \mathrm{B}$

Question13

The angle subtended by the vector $A = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$ with the X-axis is

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Options:

A. $\cos^{-1}\left(\frac{3}{13}\right)$

B. $\sin^{-1}\left(\frac{3}{13}\right)$

C. $\sin^{-1}\left(\frac{4}{13}\right)$

D. $\cos^{-1}\left(\frac{4}{13}\right)$

Answer: D

Solution:

Given, $\mathbf{A} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$

Also, $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$
$\Rightarrow A_x = 4$

The angle subtended by vector $\mathbf{A}$ with X-axis is

$$\cos\phi = \frac{A_x}{|A|} = \frac{4}{\sqrt{4^2 + 3^2 + 12^2}}$$
$$\cos\phi = \frac{4}{13}$$
$$\phi = \cos^{-1}\left(\frac{4}{13}\right)$$

Question14

What is the angle between resultant of $A + B$ and $\mathbf{A} \times \mathbf{B}$.

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Options:

A. πrad

B. 0°

C. $\frac{\pi}{4}$rad

D. $\frac{\pi}{2}$rad

Answer: D

Solution:

The vector $(A+B)$ will be in plane containing vector A and vector B.

Vector $(\mathbf{A} \times \mathbf{B})$ will be perpendicular to the plane containing vector A and B.

Thus, the angle between $(A+B)$ and $(A \times B)$ is 90° or $\frac{\pi}{2}$rad.

Alternate solution

$$
\begin{aligned}
&(A+B) \cdot(A \times B)=|A+B||A \times B| \cos \alpha \\
&A \cdot(A \times B)+B \cdot(A \times B)=|A+B||A \times B| \cos \alpha \\
&0+0=|A+B||A \times B| \cos \alpha \\
&\Rightarrow \quad \cos \alpha=0 \\
&\alpha=90^{\circ} \text { or } \frac{\pi}{2} \mathrm{rad}
\end{aligned}
$$

Question15

The x, y components of vector P have magnitudes 1 and 3 and x, y components of resultant of P and Q have magnitudes 5 and 6, respectively. What is the magnitude of Q ?

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Options:

A. 5

B. 4

C. 3

D. 2

Answer: A

Solution:

Given :

$\mathbf{P}=\hat{\mathbf{i}}+3 \hat{\mathbf{j}}$

Resultant :

$\mathbf{R} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} = \mathbf{P} + \mathbf{Q}$

Let's find the components of vector $\mathbf{Q}$:

$\Rightarrow \mathbf{Q} = \mathbf{R} - \mathbf{P}$

$\mathbf{Q} = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}}) - (\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$

$\mathbf{Q} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

To find the magnitude of $\mathbf{Q}$:

$|\mathbf{Q}| = \sqrt{(4)^2 + (3)^2}$

$|\mathbf{Q}| = \sqrt{16 + 9}$

$|\mathbf{Q}| = \sqrt{25}$

$|\mathbf{Q}| = 5$ units

Question16

The resultant of two vector A and B is C. If the magnitude of B is doubled, the new resultant vector becomes perpendicular to A. Then, the magnitude of C is

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Options:

A. 2B

B. B

C. 3B

D. 4B

Answer: B

Solution:

Given, $\mathbf{A} + \mathbf{B} = \mathbf{C}$

Resultant of A and B is

$$|C| = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta} \quad \text{... (i)}$$

When magnitude of B is doubled, new resultant (R_1) is perpendicular to A.

$$\therefore \quad |\mathbf{R}_1| \cdot \mathbf{A} = 0$$

$$|\mathbf{A} + 2\mathbf{B}| \cdot \mathbf{A} = 0$$

$$|\mathbf{A}|^2 + 2|\mathbf{A}||\mathbf{B}|\cos\theta = 0$$

$$\cos\theta = \frac{-|\mathbf{A}|}{2|\mathbf{B}|}$$

Substituting this value in Eq. (i), we get

$$|C| = \sqrt{|\mathbf{A}|^2 + |B|^2 + 2|\mathbf{A}||\mathbf{B}|\left(\frac{-|\mathbf{A}|}{2|B|}\right)}$$

$$\Rightarrow \quad |C|^2 = |\mathbf{A}|^2 - |\mathbf{A}|^2 + |\mathbf{B}|^2$$

$$|\mathbf{C}|^2 = |\mathbf{B}|^2$$

$$\therefore \quad |C| = |B| \text{ or } C = B$$

Question17

Two vectors of same magnitude have a resultant equal to either of the two vectors. The angle between two vectors is

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Options:

A. $\cos^{-1}(-0.5)$

B. $\cos^{-1}(-0.4)$

C. $\cos^{-1}(-0.3)$

D. $\cos^{-1}(-0.6)$

Answer: A

Solution:

Let's consider the two vectors to be $\mathbf{A}$ and $\mathbf{B}$, each with magnitude a. According to the question, the magnitude of the resultant vector is equal to the magnitude of either of the two vectors, i.e., resultant vector $R = a$.

To calculate the angle between the two vectors, we can use the cosine rule for vectors, which gives us the magnitude of the resultant vector R in terms of the magnitudes of $\mathbf{A}$ and $\mathbf{B}$ and the angle θ between them:

$R^2 = A^2 + B^2 + 2AB\cos\theta$

Given that $A = B = a$ and $R = a$, our equation becomes:

$a^2 = a^2 + a^2 + 2a \cdot a \cdot \cos\theta$

Simplifying it, we get:

$a^2 = 2a^2 + 2a^2\cos\theta$

$0 = a^2 + 2a^2\cos\theta$

$-1 = 2\cos\theta$

Solving for $\cos\theta$:

$\cos\theta = -\frac{1}{2}$

The angle θ whose cosine is $-\frac{1}{2}$ is $\cos^{-1}(-0.5)$. Therefore, the correct answer is:

Option A: $\cos^{-1}(-0.5)$

Question18

The vectors $(\mathbf{A}+\mathbf{B})$ and $(\mathbf{A}-\mathbf{B})$ are at right. angles to each other. This is possible under the condition

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Options:

A. $|A| = |B|$

B. $A \cdot B = 0$

C. $A \cdot B = 1$

D. $A \times B = 0$

Answer: A

Solution:

Vectors $(\mathbf{A}+\mathbf{B})$ and $(\mathbf{A}-\mathbf{B})$ are at right angle to each other, therefore

$$
\begin{aligned}
& (A+B) \cdot (A-B) = 0 \\
& A \cdot A + B \cdot A - A \cdot B - B \cdot B = 0 \\
& |A| + BA\cos\theta - AB\cos\theta - |B|^2 = 0 \\
& |A|^2 = |B|^2 \\
\text{Hence,} \quad & |A| = |B|
\end{aligned}
$$

Question19

A vector P has X and Y components of magnitude 2 units and 4 units respectively. A vector Q along negative X-axis has magnitude 6 units. The vector $(\mathbf{Q}-\mathbf{P})$ will be

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Options:

A. $4(2\hat{i}-\hat{j})$

B. $-4(2\hat{\mathbf{i}}-\hat{\mathrm{j}})$

C. $4(2\hat{i}+\hat{j})$

D. $-4(2\hat{i}+\hat{j})$

Answer: D

Solution:

According to question, vectors $\mathbf{P}$ and $\mathbf{Q}$ can be written as

$$
\begin{aligned}
& \mathbf{P} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} \\
\text{and} \quad & \mathbf{Q} = -6\hat{\mathbf{i}} \\
\therefore \quad & \mathbf{Q}-\mathbf{P} = -6\hat{\mathbf{i}} - (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \\
& = -8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} = -4(2\hat{\mathbf{i}} + \hat{\mathbf{j}})
\end{aligned}
$$

Question20

P and Q are two non-zero vectors inclined to each other at an angle ' θ '. ' p ' and ' q ' are unit vectors along P and Q respectively. The component of Q in the direction of Q will be

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Options:

A. $P \cdot Q$

B. $\frac{P \times Q}{P}$

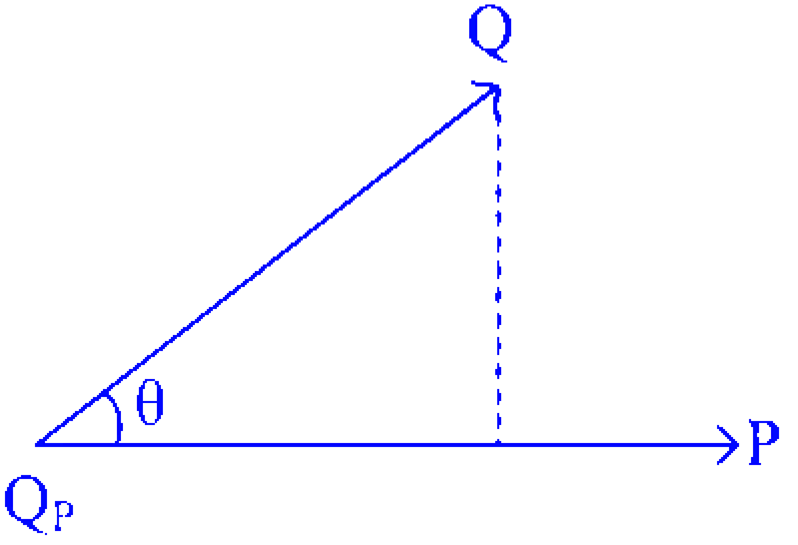
C. $\frac{P \cdot Q}{Q}$

D. $p \cdot q$

Answer: A

Solution:

Let two vectors P and Q are represented by graph as below



Here, $\mathbf{Q}_p$ is a vector in the direction of $\mathbf{P}$.

Then, from the right angle triangle, we get

$$\cos\theta = \frac{Q_p}{Q} \quad .... \text{(i)}$$

$$\Rightarrow \quad Q_p = Q\cos\theta$$

$$\text{Also, } \cos\theta = \frac{\mathbf{P}\cdot\mathbf{Q}}{PQ} \Rightarrow \frac{Q_p}{Q} = \frac{\mathbf{P}\cdot\mathbf{Q}}{PQ}$$

$$\Rightarrow \quad Q_p = \frac{\mathbf{P}\cdot\mathbf{Q}}{P} \quad .... \text{(ii)}$$

As given that, $\hat{P}$ is the unit vector along $\mathbf{P}$, then

$\hat{\mathrm{P}} = \frac{\mathrm{P}}{P}$ .... (iii)

Putting the value of P from Eq. (iii) to Eq. (ii), we get

$Q_p = \hat{\mathbf{P}}\cdot\mathbf{Q}$

Question21

The resultant R of P and Q is perpendicular to P. Also $|\mathbf{P}| = |\mathbf{R}|$. The angle between P and Q is $[\tan 45^\circ = 1]$

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Options:

A. $\frac{5\pi}{4}$

B. $\frac{7\pi}{4}$

C. $\frac{\pi}{4}$

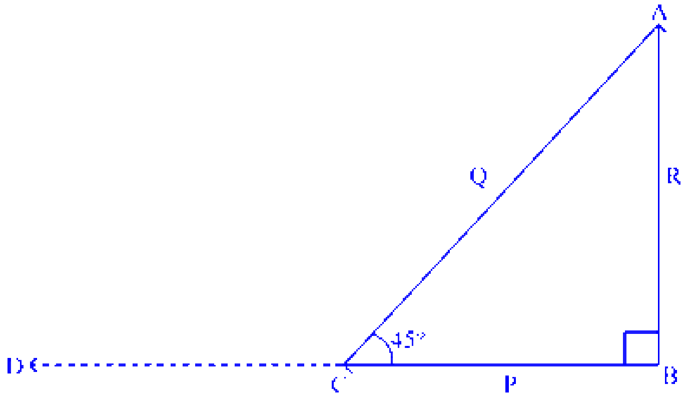
D. $\frac{3\pi}{4}$

Answer: D

Solution:

Given that $\mathbf{R}$ is resultant of $\mathbf{P}$ and $\mathbf{Q}$ as shown in the figure below,

$\mathrm{BC} = \mathrm{P}, \mathrm{CA} = Q$ and $\mathrm{BA} = R$



Given, BA and BC are perpendicular and equal in magnitude.

So, from property of triangle,

$\angle ACB = 45^\circ$

Now, BC has to be extended up to D so, that $CD = P$

Now, CD and CA have the initial point C, so the angle between CD and CA;

$= 180^\circ - 45^\circ = 135^\circ = \frac{3\pi}{4}$

So, angle between P and Q is $3\frac{\pi}{4}$.

Question22

If $\sqrt{A^2 + B^2}$ represents the magnitude of resultant of two vectors $(\mathbf{A} + \mathbf{B})$ and $(\mathbf{A} - \mathbf{B})$, then the angle between two vectors is

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Options:

A. $\cos^{-1}\left[-\frac{2(A^2-B^2)}{(A^2+B^2)}\right]$

B. $\cos^{-1}\left[-\frac{A^2-B^2}{A^2B^2}\right]$

C. $\cos^{-1}\left[-\frac{(A^2+B^2)}{2(A^2-B^2)}\right]$

D. $\cos^{-1}\left[-\frac{(A^2-B^2)}{A^2+B^2}\right]$

Answer: C

Solution:

As we know that the magnitude of the resultant of two vectors X and **Y**,

$R^2 = X^2 + Y^2 + 2XY\cos\theta$ .... (i)

where, θ is the angle between X and Y.

Putting,

$$X = (A + B)$$
$$Y = (A - B)$$

and $R = \sqrt{A^2 + B^2}$ in Eq. (i), we get

$$A^2 + B^2 = (A+B)^2 + (A-B)^2 + 2(A+B)(A-B)\cos\theta$$

$$\Rightarrow \quad A^2 + B^2 = A^2 + B^2 + 2AB + A^2$$
$$+ B^2 - 2AB + 2\left(A^2 - B^2\right)\cos\theta$$

$$\Rightarrow \frac{-\left(A^2 + B^2\right)}{2\left(A^2 - B^2\right)} = \cos\theta$$

we get, $\theta = \cos^{-1}\left[-\frac{\left(A^2 + B^2\right)}{2\left(A^2 - B^2\right)}\right]$
